

Unitarity constraints on scalar parameters of the Standard and Two Higgs Doublets Model.*

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Abstract

Unitarity constraints on the scalar parameters both for the Standard Model and the general Two Higgs Doublet Model (THDM) are examined. In the case of the THDM with an exact discrete symmetry transformation, we show that the mass of the lightest CP-even Higgs boson (M_h) and $\tan\beta$ are strongly correlated and consequently a strong lower bound can be put on M_h for large $\tan\beta$. It is also shown that the inclusion of the discrete symmetry breaking term relaxes the aforementioned correlation.

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1 Introduction

The Standard Model (SM) of electroweak interactions [1] is in complete agreement with all precision experimental data (LEP, Tevatron, SLD). What remains to be discovered is the standard Higgs boson (ϕ^0), which is the major goal of present and future searches at colliders. From the 95% CL upper limit obtained from the global Standard Model fit [2], it seems that the existence of a relatively light Higgs boson is favored ($M_{\phi^0} < 210$ GeV at 95% confidence level), which is a feature of Supersymmetric theories. If we believe in SM, the global Standard Model fit tell us that Higgs boson particle is going to be seen very soon. Direct searches at LEP show no clear evidence for the Higgs boson, and imply a lower limit $m_H > 112.3$ GeV at the 95% CL [3] (Note that this is still a preliminary limit, the previous combined limit is $m_H > 107.9$ GeV). Last September exciting new results from LEP experiments has postponed by one month the LEP program. At the end of this extra period, the data collected by the four experiments of LEP are compatible with a production of Higgs boson in association with Z boson but also compatible with other known process. The four experiment of LEP agreed that a further run with about 200 pb^{-1} per experiment at a center-of-mass energy of 208.2 GeV would be enable the four experiments to establish a 5σ discovery [4]. If there is no extra-extension for LEP run, we have to wait until either the RunII at Tevatron [5] may aid in the interpretation of the signal or the LHC, which is able to discover such a light Higgs, start up [6].

Constraints on M_{ϕ^0} can be obtained by making additional theoretical assumptions. These include i) the triviality bound [7], ii) Vacuum stability requirement and iii) demanding that unitarity of the S-matrix is not violated.

The triviality bound follows from the fact that the quartic coupling of a pure $\lambda\Phi^4$ scalar theory increases as a function of the momentum scale, causing the theory to become non-perturbative at the so-called Landau pole. The $\lambda\Phi^4$ theory will then have $\lambda(Q) = 0$ at some scale Q which renders the theory trivial.

By requiring at some large scale Λ (where new physics is supposed to enter) that the quartic coupling $\lambda(\Lambda)$ satisfies the relation $\frac{1}{\lambda(\Lambda)} > 0$ (which means that $\lambda(\Lambda)$ remains non-vanishing and so the theory is non-trivial) one can obtain an upper bound on the Higgs boson mass. For example, if this large scale Λ approaches the grand unification scale 10^{15} GeV one can find a strong approximate upper bound on the Higgs boson mass [8] $M_{\phi^0} < 160$ GeV ¹

The condition of vacuum stability requires that λ remains positive at a large scale in order to have spontaneous symmetry breaking; if λ becomes negative then the potential is unbounded from below and has no minimum. Including the 2 loop renormalisation group improved effective potential, the vacuum stability requirement gives (if the Standard model is valid up to scale 10^{16} GeV) [9, 8]: $M_{\phi^0} > 130.5 + 2.1(M_t - 174)$ GeV. Finally, unitarity arguments lead to an upper bound on the Higgs boson mass [10], and this will be covered in more detail in section 2.

In recent years there has been growing interest in the study of extended Higgs

¹The above bound changes if we couple the theory to fermions and gauge bosons.

sectors with more than one doublet [11]. The simplest extension of the MSM is the Two Higgs Doublet Model (THDM), which is formed by adding an extra complex $SU(2)_L \otimes U(1)_Y$ scalar doublet to the MSM Lagrangian. Motivations for such a structure include CP-violation in the Higgs sector, supersymmetry, and a possible solution to the cosmological domain wall problem [12]. In particular, the Minimal Supersymmetric Standard Model (MSSM) [11] takes the form of a constrained THDM. The two most popular versions of THDM are classified as type I and type II, differing in how the Higgs bosons are coupled to the fermions although both versions possess identical particle spectra after electroweak symmetry breaking. From the eight degrees of freedom initially present in the two Higgs doublets, three correspond to masses of the longitudinal gauge bosons, leaving five degrees of freedom which manifest themselves as five physical Higgs particles: Two charged Higgs H^\pm , two CP-even H^0 , h^0 and one CP-odd A^0 . Until now no Higgs boson has been discovered, and from the null searches one can derive direct and indirect bounds on their masses. The latest such limits are: for the charged Higgs boson the LEP combined result is $m_{H^\pm} > 77.5$ GeV [3], and for the neutral Higgs OPAL collaboration [13] has made the following scan of the parameter space as: $1 < m_h < 100$, $5 < m_A < 2$ TeV, $-\pi/2 < \alpha < 0$ and $0.4 < \tan \beta < 58$ finding that the region ($1 < m_h < 44$ GeV and $12 < m_A < 56$ GeV) is excluded at 95% independent of α and $\tan \beta$.

We note here that tree-level unitarity constraints for the THDM scalar potential constrained by the exact discrete symmetry $\Phi_i \rightarrow -\Phi_i$ were studied in [14, 15]. When deriving constraints from unitarity, [14] considered only seven elastic scattering processes $S_1 S_2 \rightarrow S_1 S_2$ (where S_i is a Higgs scalar) while [15] considered a larger scattering (S) matrix. In [15], upper bounds on the Higgs masses were derived, in particular $M_h \leq 410$ GeV for $\tan \beta = 1$, with the bound becoming stronger as $\tan \beta$ increases. In section. 3, at the same footing as we will present for the MSM in section. 2, we improve the aforementioned THDM studies by including the full scalar S matrix which includes channels which were absent in [15]. In addition, we also show graphically the strong correlation between M_h and $\tan \beta$ both in the case where the discrete symmetry $\Phi_i \rightarrow -\Phi_i$ is exact and also in the case where it is broken by dimension 2 terms: $\lambda_5 \Re(\Phi_1^\dagger \Phi_2)$.

The paper is organized as follows. In Section 2 we present the unitarity approach we will be using and review the unitarity constraints in the framework of the MSM. Section 3 contains a short review of the THDM potential and the analytical unitarity constraints. In Section 4 we present our numerical results for the cases of $\lambda_5 = 0$ and $\neq 0$, while Section 5 contains our conclusions. The appendix contains pure scalar quartic interactions both for MSM and for THDM.

2. Unitarity approach

We will study the unitarity constraints by computing the scalar scattering processes, $S_1 S_2 \rightarrow S_3 S_4$, i.e. both elastic and inelastic channels. In terms of the partial wave

decomposition, the amplitude \mathcal{M} of a scattering $S_1 S_2 \rightarrow S_3 S_4$ can be written as:

$$\mathcal{M}(s, t, u) = 16\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos \theta) a_l(s) \quad (2.1)$$

Where s, t, u are the Mandelstam variables, $a_l(s)$ is the spin l partial wave and P_l are Legendre Polynomials. The differential cross section for $S_1 S_2 \rightarrow S_3 S_4$ is given by:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |\mathcal{M}|^2$$

Using the fact the the Legendre polynomials are orthogonal, the cross section becomes:

$$\sigma = \frac{16\pi}{s} \sum_{l=0}^{\infty} (2l+1) |a_l|^2$$

The optical theorem together with expression for the total cross section given above leads to the following unitarity constraint:

$$\Re(a_l)^2 + \Im(a_l)^2 = |a_l|^2 = \Im(a_l) \quad \text{for all } l$$

The above equation is nothing more than an equation for a circle in the plane $(\Re(a_l), \Im(a_l))$ with radius $\frac{1}{2}$ and center $(0, \frac{1}{2})$. It can be shown easily from the graphical representation of this circle that:

$$|\Re(a_l)| < \frac{1}{2} \quad \text{for all } l$$

The partial wave $a_l(s)$ can be inversed from eq.2.1 and one finds:

$$a_l(s) = \frac{1}{32\pi} \int_{-1}^1 d(\cos \theta) P_l(\cos \theta) \mathcal{M}(s, t, u)$$

If we limit ourselves to the $J = 0$ s-wave amplitude $a_0(s)$, for vanishing external masses of $S_{1,2,3,4}$, $a_0(s)$ take the following form:

$$a_0(s) = \frac{1}{16\pi} [Q + \{T_h^{12} T_h^{34} \frac{1}{s - M_h^2} - \frac{1}{s} (c_t T_h^{13} T_h^{24} + c_u T_h^{14} T_h^{23}) \ln(1 + \frac{s}{m_h^2})\}] \quad (2.2)$$

Where Q is the four point vertex for $S_1 S_2 \rightarrow S_3 S_4$ and T_h^{ij} are the trilinear vertex $h S_i S_j$, $c_t = 1$ or 0 (resp $c_u = 1$ or 0) for processes with or without t-channel (resp for process with or without u-channel).

The first term in the bracket in the r.h.s of eq.2.2 is the contribution of the quartic coupling $Q = S_1 S_2 S_3 S_4$ to the amplitude \mathcal{M} ; the second term is the contribution of the s-channel diagram with exchange of particle h and the third term is the contribution of t and u channels.

In very high energy collisions, it can be shown from eq.2.2 that the dominant contribution to the amplitude of the two-body scattering $S_1 S_2 \rightarrow S_3 S_4$ is the one which is mediated by the quartic coupling. Those contributions mediated by trilinear couplings are suppressed on dimensional grounds. Therefore the unitarity

constraint $|a_0| \leq 1/2$ reduces to the following constraint on the quartic coupling, $|Q(S_1 S_2 S_3 S_4)| \leq 8\pi$. In what follows our attention will be focused on the quartic couplings.

To constrain the scalar potential parameters one can demand that tree-level unitarity is preserved in a variety of scattering processes. This corresponds to the requirement that the $J = 0$ partial waves (a_0) for scalar-scalar, gauge boson-gauge boson and gauge boson-scalar scattering satisfy $|a_0| < 1/2$ in the high-energy limit. At very high-energy, the equivalence theorem [16] states that the amplitude of a scattering process involving longitudinal gauge bosons $V_\mu^{\pm,0}$ may be approximated by the scalar amplitude in which gauge bosons are replaced by their corresponding Goldstone bosons $G^{\pm,0}$. We conclude that unitarity constraints can be implemented by solely considering pure scalar scattering.

Let us apply all these features to the non-coupled process $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$ in the MSM, where the subscript L denotes the longitudinal polarisation states. As stated above, at high energy the amplitude of $W_L^+ W_L^- \rightarrow W_L^+ W_L^-$ is approximated by $G^+ G^- \rightarrow G^+ G^-$ (where G^\pm is the charged goldstone associated with W^\pm) whose dominate contribution comes the quartic coupling, $G^+ G^- G^+ G^- = M_{\phi^0}^2/v^2$, where v is fixed by the electroweak scale as $v^2 = (2\sqrt{2}G_F)^{-1}$. The unitarity constraint $|Q| \leq 8\pi$ gives the following upper bound on the Higgs boson's mass:

$$M_{\phi^0}^2 < 8\pi v^2 = M_{LQT}^2 = \frac{4\pi}{\sqrt{2}G_F} = (870)^2 \text{GeV}^2$$

Where M_{LQT} is the bound deduced by Lee, Quigg and Thacker [10].

In fact the channel $W_L^+ W_L^-$ considered above is coupled with the following channels: $Z_L Z_L/\sqrt{2}$, $\phi^0 \phi^0/\sqrt{2}$ and $Z_L \phi^0$ (the factor $\sqrt{2}$ accounts for identical particle statistics). Together with those four neutral initial states we consider also the 2 charged channels $W_L^+ \phi^0$ and $W_L^+ Z$. Note that due to charge conservation the two charged channels are not coupled to the four charged channels. Taking into account all the above channels, the scattering amplitude is given by 6×6 matrix which is diagonal by block: 4×4 block for the 4 neutral channels and 2×2 block for 2 charged channels. At high energies, the matrix elements are dominated by the quartic couplings and so the full matrix in the basis $(W_L^+ W_L^-, Z_L Z_L/\sqrt{2}, \phi^0 \phi^0/\sqrt{2}, Z_L \phi^0, W_L^+ \phi^0, W_L^+ Z)$ takes the following form:

$$a_0 = \frac{M_{\phi^0}^2}{v^2} \begin{pmatrix} 1 & \frac{\sqrt{2}}{4} & \frac{\sqrt{2}}{4} & 0 & 0 & 0 \\ \frac{\sqrt{2}}{4} & \frac{3}{4} & \frac{1}{4} & 0 & 0 & 0 \\ \frac{\sqrt{2}}{4} & \frac{1}{4} & \frac{3}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{pmatrix} \quad (2.3)$$

This matrix can be read easily from the appendix A.1 in which we have listed the pure scalar quartic interactions in the MSM.

The requirement that the largest eigenvalues² of a_0 respects the unitarity constraint

²In the case of this matrix the largest eigenvalues is $3/2$

yields[17]:

$$M_{\phi^0}^2 < \frac{2}{3}8\pi v^2 = \frac{2}{3}M_{LQT}^2 \approx (710)^2 \text{GeV}^2$$

We conclude that the inclusion of the complete set of scattering channels (charged and neutral ones) into the analysis, leads more or less to a stronger unitarity constraint on the Higgs boson mass $M_{\phi^0} < 710 \text{GeV}$. The unitarity requirement tells us that if the Higgs boson mass is above the unitarity constraint then the standard model becomes non-perturbative and some new physics should appear at some high scale to restore the unitarity of the theory.

3. THDM scalar potential and Unitarity constraints

3.1 THDM scalar potential

The most general THDM scalar potential which is renormalizable, gauge invariant and CP invariant depends on ten parameters, but such a potential can still break CP spontaneously [18]. In order to ensure that tree-level flavor changing neutral currents are eliminated, a discrete symmetry ($\Phi_i \rightarrow -\Phi_i$, where Φ_i is a scalar doublet) may be imposed on the lagrangian [19], which reduces the number of free parameters to 6. The resulting potential was considered in [20], and is referred to as V_A in [21]. We shall be concerned with the potential described in [11] which is equivalent to V_A plus a term which breaks the discrete symmetry (parameterized by λ_5) and contains 7 free parameters. Such a potential does not break CP spontaneously or explicitly [20],[22] provided that all the parameters are real.

It has been shown [23] that the most general THDM scalar potential which is invariant under $SU(2)_L \otimes U(1)_Y$ and conserves CP is given by:

$$\begin{aligned} V(\Phi_1, \Phi_2) &= \lambda_1(|\Phi_1|^2 - v_1^2)^2 + \lambda_2(|\Phi_2|^2 - v_2^2)^2 + \lambda_3((|\Phi_1|^2 - v_1^2) + (|\Phi_2|^2 - v_2^2))^2 + \\ &\quad \lambda_4(|\Phi_1|^2|\Phi_2|^2 - |\Phi_1^+\Phi_2|^2) + \lambda_5(\Re(\Phi_1^+\Phi_2) - v_1v_2)^2 + \lambda_6[\Im(\Phi_1^+\Phi_2)]^2 \end{aligned} \quad (3.1)$$

where Φ_1 and Φ_2 have weak hypercharge $Y=1$, v_1 and v_2 are respectively the vacuum expectation values of Φ_1 and Φ_2 and the λ_i 's are real-valued parameters.

$$\Phi_i = \begin{pmatrix} w_i^+ \\ v_i + \frac{h_i + iz_i}{\sqrt{2}} \end{pmatrix}$$

This potential violates the discrete symmetry $\Phi_i \rightarrow -\Phi_i$ softly by the dimension 2 term $\lambda_5 \Re(\Phi_1^+\Phi_2)$ and has the same general structure of the scalar potential of the MSSM. One can prove easily that for $\lambda_5 = 0$ the exact symmetry $\Phi_i \rightarrow -\Phi_i$ is recovered.

After electroweak symmetry breaking, the W and Z gauge bosons acquire masses given by $m_W^2 = \frac{1}{2}g^2v^2$ and $m_Z^2 = \frac{1}{2}(g^2 + g'^2)v^2$, where g and g' are the $SU(2)_{weak}$ and $U(1)_Y$ gauge couplings and $v^2 = v_1^2 + v_2^2$. The combination $v_1^2 + v_2^2$ is thus fixed by the electroweak scale through $v_1^2 + v_2^2 = (2\sqrt{2}G_F)^{-1}$, and we are left with 7 free parameters in eq.(3.1), namely the $(\lambda_i)_{i=1,\dots,6}$'s and $\tan \beta = v_2/v_1$. Meanwhile,

three of the eight degrees of freedom of the two Higgs doublets correspond to the 3 Goldstone bosons (G^\pm , G^0) and the remaining five become physical Higgs bosons: H^0 , h^0 (CP-even), A^0 (CP-odd) and H^\pm . Their masses are obtained as usual by the shift $\Phi_i \rightarrow \Phi_i + v_i$ [11] and read

$$m_{A^0}^2 = \lambda_6 v^2 ; \quad m_{H^\pm}^2 = \lambda_4 v^2 \quad \text{and} \quad m_{H^0, h^0}^2 = \frac{1}{2}[A + C \pm \sqrt{(A - C)^2 + 4B^2}] \quad (3.2)$$

where

$$A = 4v_1^2(\lambda_1 + \lambda_3) + v_2^2\lambda_5 \quad , \quad B = v_1v_2(4\lambda_3 + \lambda_5) \quad \text{and} \quad C = 4v_2^2(\lambda_2 + \lambda_3) + v_1^2\lambda_5$$

The angle β given by $\tan \beta = v_2/v_1$ defines the mixing leading to the physical H^\pm and A^0 states, while the mixing angle α associated to H^0, h^0 physical states is given by (See [11] for details.)

$$\sin 2\alpha = \frac{2B}{\sqrt{(A - C)^2 + 4B^2}} \quad , \quad \cos 2\alpha = \frac{A - C}{\sqrt{(A - C)^2 + 4B^2}} \quad (3.3)$$

It will be more suitable for the forthcoming discussion to trade the five parameters $\lambda_{1,2,4,5,6}$ for the 4 Higgs masses and the mixing angle α . From now on we will take the physical Higgs masses, $m_{H^0}, m_{h^0}, m_{A^0}, m_{H^\pm}$, the mixing angles α, β and the coupling λ_5 as the 7 free parameters.

It is then straightforward algebra to invert equations (3.2) through (3.3), and get the λ_i 's in terms of this new set of parameters.

$$\lambda_4 = \frac{g^2}{2m_W^2} m_{H^\pm}^2 \quad , \quad \lambda_6 = \frac{g^2}{2m_W^2} m_{A^0}^2 \quad , \quad \lambda_3 = \frac{g^2}{8m_W^2} \frac{s_\alpha c_\alpha}{s_\beta c_\beta} (m_H^2 - m_h^2) - \frac{\lambda_5}{4} \quad (3.4)$$

$$\lambda_1 = \frac{g^2}{8c_\beta^2 m_W^2} [c_\alpha^2 m_H^2 + s_\alpha^2 m_h^2 - \frac{s_\alpha c_\alpha}{\tan \beta} (m_H^2 - m_h^2)] - \frac{\lambda_5}{4} (-1 + \tan^2 \beta) \quad (3.5)$$

$$\lambda_2 = \frac{g^2}{8s_\beta^2 m_W^2} [s_\alpha^2 m_H^2 + c_\alpha^2 m_h^2 - s_\alpha c_\alpha \tan \beta (m_H^2 - m_h^2)] - \frac{\lambda_5}{4} (-1 + \frac{1}{\tan^2 \beta}) \quad (3.6)$$

3.2 Unitarity constraints

To constrain the scalar potential parameters of the THDM one can demand that tree-level unitarity is preserved in a variety of scattering processes: scalar-scalar scattering, gauge boson-gauge boson scattering and scalar-gauge boson scattering. We will follow exactly the technique developed in section.2 and therefore we limit ourselves to pure scalar scattering processes dominated by quartic interactions.

In order to derive the unitarity constraints on the scalar masses we will adopt the technique introduced in [15]. It has been shown in previous works [25] that the quartic scalar vertices written in terms of physical fields $H^\pm, G^\pm, h^0, H^0, A^0$ and G^0 , are very complicated functions of λ_i, α and β . However the quartic vertices (computed before electroweak symmetry breaking) written in terms of the non-physical

fields w_i^\pm , h_i and z_i ($i=1,2$) are considerably simpler expressions. The crucial point of [15] is the fact that the S matrix expressed in terms of the physical fields (i.e. the mass eigenstate fields) can be transformed into an S matrix for the non-physical fields φ_i^\pm , h_i and z_i by making a unitarity transformation. The latter is relatively easy to compute from eq. 3.1. Therefore the full set of scalar scattering processes can be expressed as an S matrix 22×22 composed of 4 submatrices [$\mathcal{M}_1(6 \times 6)$, $\mathcal{M}_2(6 \times 6)$, $\mathcal{M}_3(6 \times 6)$ and $\mathcal{M}_4(8 \times 8)$] which do not couple with each other due to charge conservation and CP-invariance. The entries are the quartic couplings which mediate the scattering processes. The pure scalar quartic interactions expressed in terms of the non-physical fields h_i^\pm , h_i and z_i are listed in the appendix A.2.

The first submatrix \mathcal{M}_1 corresponds to scattering whose initial and final states are one of the following: $(w_1^+ w_2^-, w_2^+ w_1^-, h_1 z_2, h_2 z_1, z_1 z_2, h_1 h_2)$. With the help of appendix A.2, one can find that \mathcal{M}_1 takes the following form:

$$\mathcal{M}_1 = \begin{pmatrix} 2\lambda_3 + \lambda_{56}^+/2 & \lambda_{56}^- & -i\lambda_{64}^-/2 & i\lambda_{64}^-/2 & \lambda_{54}^-/2 & \lambda_{54}^-/2 \\ \lambda_{56}^- & 2\lambda_3 + \lambda_{56}^+/2 & i\lambda_{64}^-/2 & -i\lambda_{64}^-/2 & \lambda_{54}^-/2 & \lambda_{54}^-/2 \\ i\lambda_{64}^-/2 & -i\lambda_{64}^-/2 & 2\lambda_3 + \lambda_6 & \lambda_{56}^-/2 & 0 & 0 \\ -i\lambda_{64}^-/2 & i\lambda_{64}^-/2 & \lambda_{56}^-/2 & 2\lambda_3 + \lambda_6 & 0 & 0 \\ \lambda_{54}^-/2 & \lambda_{54}^-/2 & 0 & 0 & 2\lambda_3 + \lambda_5 & \lambda_{56}^-/2 \\ \lambda_{54}^-/2 & \lambda_{54}^-/2 & 0 & 0 & \lambda_{56}^-/2 & 2\lambda_3 + \lambda_5 \end{pmatrix}$$

where $\lambda_{ij}^\pm = \lambda_i \pm \lambda_j$. With the help of Mathematica we find that \mathcal{M}_1 has the following 5 distinct eigenvalues:

$$\begin{aligned} e_1 &= 2\lambda_3 - \lambda_4 - \frac{\lambda_5}{2} + \frac{5}{2}\lambda_6 \\ e_2 &= 2\lambda_3 + \lambda_4 - \frac{\lambda_5}{2} + \frac{1}{2}\lambda_6 \\ f_+ &= 2\lambda_3 - \lambda_4 + \frac{5}{2}\lambda_5 - \frac{1}{2}\lambda_6 \\ f_- &= 2\lambda_3 + \lambda_4 + \frac{1}{2}\lambda_5 - \frac{1}{2}\lambda_6 \\ f_1 &= f_2 = 2\lambda_3 + \frac{1}{2}\lambda_5 + \frac{1}{2}\lambda_6 \end{aligned} \quad (3.7)$$

The second submatrix \mathcal{M}_2 corresponds to scattering with initial and final states one of the following: $(w_1^+ w_1^-, w_2^+ w_2^-, \frac{z_1 z_1}{\sqrt{2}}, \frac{z_2 z_2}{\sqrt{2}}, \frac{h_1 h_1}{\sqrt{2}}, \frac{h_2 h_2}{\sqrt{2}})$, where the $\sqrt{2}$ accounts for identical particle statistics. Again, with the help of appendix A.2, one find that \mathcal{M}_2 is given by:

$$\mathcal{M}_2 = \begin{pmatrix} 4\lambda_{13}^+ & 2\lambda_3 + \lambda_{56}^+/2 & \sqrt{2}\lambda_{13}^+ & \tilde{\lambda}_{34}/\sqrt{2} & \sqrt{2}\lambda_{13}^+ & \tilde{\lambda}_{34}/\sqrt{2} \\ 2\lambda_3 + \frac{\lambda_{56}^+}{2} & 4\lambda_{23}^+ & \tilde{\lambda}_{34}/\sqrt{2} & \sqrt{2}\lambda_{23}^+ & \tilde{\lambda}_{34}/\sqrt{2} & \sqrt{2}\lambda_{23}^+ \\ \sqrt{2}\lambda_{13}^+ & \tilde{\lambda}_{34}/\sqrt{2} & 3\lambda_{13}^+ & \tilde{\lambda}_{35}/2 & \lambda_{13}^+ & \tilde{\lambda}_{36}/2 \\ \tilde{\lambda}_{34}/\sqrt{2} & \sqrt{2}\lambda_{23}^+ & \tilde{\lambda}_{35}/2 & 3\lambda_{23}^+ & \tilde{\lambda}_{36}/2 & \lambda_{23}^+ \\ \sqrt{2}\lambda_{13}^+ & \tilde{\lambda}_{34}/\sqrt{2} & \lambda_{13}^+ & \tilde{\lambda}_{36}/2 & 3\lambda_{13}^+ & \tilde{\lambda}_{35}/2 \\ \tilde{\lambda}_{34}/\sqrt{2} & \sqrt{2}\lambda_{23}^+ & \tilde{\lambda}_{36}/2 & \lambda_{23}^+ & \tilde{\lambda}_{35}/2 & 3\lambda_{23}^+ \end{pmatrix} \quad (3.8)$$

where $\tilde{\lambda}_{3i} = 2\lambda_3 + \lambda_i$. The matrix \mathcal{M}_2 possesses the following 6 distinct eigenvalues:

$$a_{\pm} = 3(\lambda_1 + \lambda_2 + 2\lambda_3) \pm \sqrt{9(\lambda_1 - \lambda_2)^2 + (4\lambda_3 + \lambda_4 + \frac{1}{2}(\lambda_5 + \lambda_6))^2} \quad (3.9)$$

$$b_{\pm} = \lambda_1 + \lambda_2 + 2\lambda_3 \pm \sqrt{(\lambda_1 - \lambda_2)^2 + \frac{1}{4}(-2\lambda_4 + \lambda_5 + \lambda_6)^2} \quad (3.10)$$

$$c_{\pm} = \lambda_1 + \lambda_2 + 2\lambda_3 \pm \sqrt{(\lambda_1 - \lambda_2)^2 + \frac{1}{4}(\lambda_5 - \lambda_6)^2} \quad (3.11)$$

The third submatrix \mathcal{M}_3 corresponds to the basis: $(h_1 z_1, h_2 z_2)$ and is given by:

$$\mathcal{M}_3 = \begin{pmatrix} 2\lambda_{13}^+ & \frac{1}{2}\lambda_{56}^- \\ \frac{1}{2}\lambda_{56}^- & 2\lambda_{23}^+ \end{pmatrix} \quad (3.12)$$

The matrix \mathcal{M}_3 possesses the eigenvalues d_{\pm} and c_{\pm} , with $d_{\pm} = c_{\pm}$. All the above eigenvalues agree with those found in [15], up to a factor of $1/16\pi$ which we have factorised out. In our analysis we also include the two body scattering between the 8 charged states: $h_1 w_1^+$, $h_2 w_1^+$, $z_1 w_1^+$, $z_2 w_1^+$, $h_1 w_2^+$, $h_2 w_2^+$, $z_1 w_2^+$, $z_2 w_2^+$. The 8×8 submatrix \mathcal{M}_4 obtained from the above scattering processes is given by:

$$\mathcal{M}_4 = \begin{pmatrix} 2\lambda_{13}^+ & 0 & 0 & 0 & 0 & \lambda_{54}^-/2 & 0 & -i\lambda_{64}^-/2 \\ 0 & 2\lambda_3 + \lambda_4 & 0 & 0 & \frac{1}{2}\lambda_{54}^- & 0 & i\frac{1}{2}\lambda_{64}^- & 0 \\ 0 & 0 & 2\lambda_{13}^+ & 0 & 0 & \frac{i}{2}\lambda_{64}^- & 0 & \lambda_{54}^-/2 \\ 0 & 0 & 0 & 2\lambda_3 + \lambda_4 & -\frac{i}{2}\lambda_{64}^- & 0 & \frac{1}{2}\lambda_{54}^- & 0 \\ 0 & \frac{1}{2}\lambda_{54}^- & 0 & \frac{i}{2}\lambda_{64}^- & 2\lambda_3 + \lambda_4 & 0 & 0 & 0 \\ \frac{1}{2}\lambda_{54}^- & 0 & -\frac{i}{2}\lambda_{64}^- & 0 & 0 & 2\lambda_{23}^+ & 0 & 0 \\ 0 & -\frac{i}{2}\lambda_{64}^- & 0 & \frac{1}{2}\lambda_{54}^- & 0 & 0 & 2\lambda_3 + \lambda_4 & 0 \\ \frac{i}{2}\lambda_{64}^- & 0 & \frac{1}{2}\lambda_{54}^- & 0 & 0 & 0 & 0 & 2\lambda_{23}^+ \end{pmatrix}$$

As one can see, this matrix contains many vanishing elements, and the 8 eigenvalues are straightforward to obtain analytically. They read as follows: f_- , e_2 , f_1 , c_{\pm} , b_{\pm} and p_1 , where

$$p_1 = 2(\lambda_3 + \lambda_4) - \frac{1}{2}\lambda_5 - \frac{1}{2}\lambda_6 \quad (3.13)$$

As one can see, these additional channels lead only to one extra eigenvalue, p_1 , although we shall see that this eigenvalue plays an important role in constraining $M_{H^{\pm}}$ and M_A .

4. Numerical results and discussion

In this section we present our results for the unitarity constraints on the Higgs masses in the THDM. All the eigenvalues are constrained as follows:

$$|a_{\pm}|, |b_{\pm}|, |c_{\pm}|, |d_{\pm}|, |f_{\pm}|, |e_{1,2}|, |f_{1,2}|, |p_1| \leq 8\pi \quad (4.1)$$

In our numerical illustrations, we will consider the special case of $\lambda_5 = 0$ and also the general case of $\lambda_5 \neq 0$.

Case of $\lambda_5 = 0$

For $\lambda_5 = 0$ our potential is identical to those considered in [14, 15]. We improve those analyses on two accounts:

- (i) We have considered extra scattering channels which leads to one more eigenvalue constraint, p_1 , as explained in Section. 3.
- (ii) When finding the allowed parameter space of Higgs masses we simultaneously impose all the eigenvalue constraints. In [15] only the condition $|a_+| \leq 8\pi$ was applied when deriving mass bounds.

In order to obtain the upper bounds on the Higgs masses allowed by the unitarity constraints we vary all the Higgs masses and mixing angles randomly over a very large parameter space. As is reported in [24], we confirm the result of [15] which states that a_+ is comfortably the strongest individual eigenvalue constraint. However, the other eigenvalues impose important constraints on M_A and $M_{H^{\pm}}$. If only $|a_+| \leq 8\pi$ is imposed we can reproduce the upper bounds on the Higgs masses given in [15], in particular their main result of $M_h \leq 410$ GeV. When all eigenvalue bounds are applied simultaneously we find improved bounds on the Higgs masses, particularly for M_A and $M_{H^{\pm}}$. We note that the new eigenvalue constraint $p_1 \leq 8\pi$ (eq.3.13) plays a crucial role in determining the upper bound on $M_{H^{\pm}}$. For $\lambda_5 = 0$, we find the following upper bounds:

$$M_{H^{\pm}} < 691 \quad , \quad M_A < 695 \quad , \quad M_h < 435 \quad , \quad M_H < 638 \quad (GeV)$$

Note that the bounds given above are obtained for relatively small $\tan\beta$ (say $\tan\beta \approx 0.5$). For large $\tan\beta$ the bound is stronger, although for the case of A^0 , H^0 and H^{\pm} the $\tan\beta$ dependence is rather gentle. Of particular interest is the $\tan\beta$ dependence of the bound on M_h which will be covered below.

We have to stress here that our results in the case where the discrete symmetry is exact agree with the results obtained using triviality approach [28].

General case of $\lambda_5 \neq 0$

We now consider $\lambda_5 \neq 0$ which corresponds to the inclusion of the term which softly breaks the discrete symmetry. Such a term was neglected in the analyses of

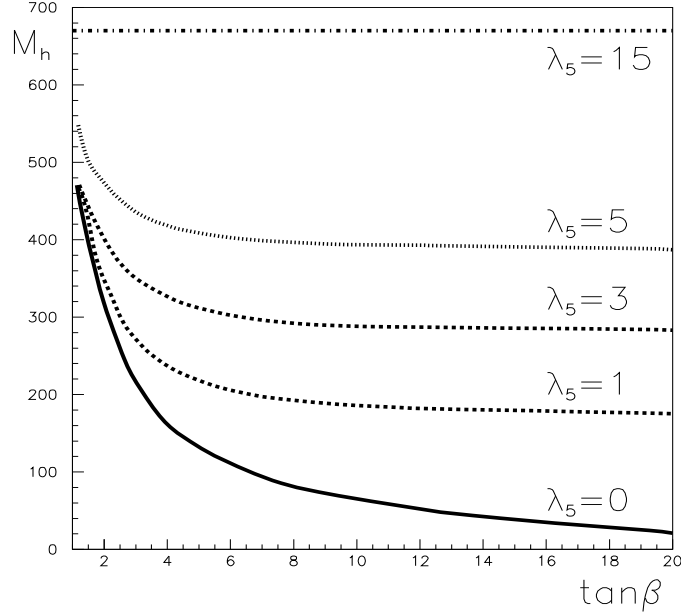


Figure 1: Maximum M_h in GeV as a function of $\tan \beta$ for various values of λ_5 .

[14, 15], and from perturbative constraints may take values $|\lambda_5| \leq 8\pi$ [26]. In the graphs which follow we do not impose the perturbative requirement $|\lambda_i| \leq 8\pi$ for the remaining λ_i . Imposing this condition only leads to minor changes in the numerical results which will be commented on when necessary.

We plot in Fig.1 the maximum value of M_h against $\tan \beta$ for increasing values of λ_5 , imposing all the eigenvalue constraints simultaneously as done in Section 3.1. For the case of $\lambda_5 = 0$ one finds a strong correlation, with larger $\tan \beta$ requiring smaller M_h . For example, $\tan \beta \geq 7$ corresponds to $M_h \leq 100$ GeV, which is the mass range already being probed by LEP II. However, h^0 in the THDM with $M_h \leq 100$ GeV is not guaranteed to be found at LEP II due to the suppression factor of $\sin^2(\beta - \alpha)$ for the main production process $e^+e^- \rightarrow h^0 Z$. For the case of $\lambda_5 = 0$ we find that values of $\tan \beta \geq 20$ are strongly disfavoured since they easily violate one of the unitarity constraints. If $\lambda_5 \neq 0$, Fig.1 shows that for a given $\tan \beta$, the action of increasing λ_5 allows larger maximum values of M_h . For $\lambda_5 = 15$ one finds a horizontal line at $M_h \approx 670$ GeV, showing that the upper bound has been increased for all values of $\tan \beta$. In addition, large (≥ 30) values of $\tan \beta$ are allowed if $\lambda_5 \neq 0$, in contrast to the case of $\lambda_5 = 0$. For example, $\lambda_5 = 1$ comfortably permits values of $\tan \beta = 60$. However, as pointed out in [27] perturbative constraints on the λ_i also restrict the allowed values of $\tan \beta$ in the THDM. Using the condition in [26] which requires $|\lambda_i| \leq 8\pi$, we found that $\tan \beta \geq 30$ is strongly disfavoured.

We note that the relaxation of the strong correlation between M_h and $\tan \beta$ with $\lambda_5 \neq 0$ would in principle allow the possibility of distinguishing between the

discrete symmetry conserving and violating potentials. If h^0 is discovered and the measured values of M_h and $\tan\beta$ lie outside the rather constrained region for $\lambda_5 = 0$, this would signify $\lambda_5 \neq 0$ and thus a soft breaking of the discrete symmetry.

5. Conclusions

We have derived upper limits on the masses of the Higgs bosons both in the Minimal Standard Model (MSM) and also in the general Two Higgs Doublet Model (THDM) by requiring that unitarity is not violated in a variety of scattering processes.

In the MSM we have revived the unitarity constraints and shown that the inclusion of all the coupled scattering channels in the analysis can make the upper limit on the Higgs boson stronger than in the non-coupled case. The same analysis has been done for the THDM, where we first considered the THDM scalar potential which is invariant under a discrete symmetry transformation and improved previous studies by including the complete set of scattering channels. Stronger constraints on the Higgs masses were derived. Of particular interest is the $\tan\beta$ dependence of the upper bound on M_h , with larger $\tan\beta$ requiring a lighter h^0 e.g. $\tan\beta \geq 7$ implies $M_h \leq 100$ GeV. We then showed that the presence of the discrete symmetry breaking term parametrized by λ_5 may significantly weaken the upper bounds on the masses. In particular, the aforementioned correlation between $\tan\beta$ and maximum M_h is relaxed. It was suggested that a measurement of $\tan\beta$ and M_h may allow discrimination between the two potentials.

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Appendix A.

A.1 Quartic scalar interactions in the MSM

The pure quartic scalar interactions in the MSM can be found from the scalar potential $V = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$ where $\Phi = \begin{pmatrix} G^\pm \\ v + (\phi^0 + iG^0)/\sqrt{2} \end{pmatrix}$, G^\pm and G^0 are the goldstone boson and ϕ^0 is the physical Higgs. Keeping only the quartic terms in V we get the following Feynman rules:

$$\begin{aligned} G^+G^-G^+G^- &= \frac{M_{\phi^0}^2}{v^2} \quad , \quad G^+G^-G^0G^0 = \frac{3}{2}\frac{M_{\phi^0}^2}{v^2} \quad , \quad G^+G^-\phi_0\phi_0 = \frac{M_{\phi^0}^2}{2v^2} \\ G^0G^0G^0G^0 &= \frac{3}{2}\frac{M_{\phi^0}^2}{v^2} \quad , \quad \phi_0\phi_0G^0G^0 = \frac{M_{\phi^0}^2}{2v^2} \quad , \quad \phi_0\phi_0\phi_0\phi_0 = \frac{3}{2}\frac{M_{\phi^0}^2}{v^2} \end{aligned}$$

where v^2 is fixed by the electroweak scale by $v^2 = (2\sqrt{2}G_F)^{-1}$, $M_{\phi^0}^2 = 4v^2\lambda$ and $v^2 = \frac{\mu^2}{4\lambda}$.

A.2 Quartic scalar interactions in the THDM

We start from the potential 3.1 in which the Higgs doublets are expressed in terms of the non-physical fields:

$$\Phi_i = \begin{pmatrix} w_i^+ \\ v_i + \frac{1}{\sqrt{2}}(h_i + iz_i) \end{pmatrix}$$

Expanding this potential and keeping only the quartic terms leads to the following Feynman rules:

$$\begin{aligned} w_1^+w_1^-w_1^+w_1^- &= 4(\lambda_1 + \lambda_3) \quad , & w_1^+w_1^+w_2^-w_2^- &= (\lambda_5 - \lambda_6) \\ w_1^-w_1^+w_2^-w_2^+ &= 2\lambda_3 + \frac{1}{2}(\lambda_5 + \lambda_6) \quad , & w_1^-w_1^-w_2^+w_2^+ &= (\lambda_5 - \lambda_6) \\ w_2^+w_2^-w_2^+w_2^- &= 4(\lambda_2 + \lambda_3) \quad , & h_1h_1h_1h_1 &= 6(\lambda_1 + \lambda_3) \\ h_2h_2h_2h_2 &= 6(\lambda_2 + \lambda_3) \quad , & h_1h_1h_2h_2 &= 2\lambda_3 + \lambda_5 \\ h_1h_1w_1^-w_1^+ &= 2(\lambda_1 + \lambda_3) \quad , & h_2h_2w_1^-w_1^+ &= 2\lambda_3 + \lambda_4 \\ h_1h_2w_1^+w_2^- &= (\lambda_5 - \lambda_4)/2 \quad , & h_2h_2w_2^-w_2^+ &= 2(\lambda_2 + \lambda_3) \\ h_1h_2w_1^-w_2^+ &= (\lambda_5 - \lambda_4)/2 \quad , & h_1h_1w_2^-w_2^+ &= 2\lambda_3 + \lambda_4 \\ z_2h_1w_1^+w_2^- &= \frac{i}{2}(\lambda_6 - \lambda_4) \quad , & z_2z_2w_1^+w_1^- &= 2\lambda_3 + \lambda_4 \end{aligned}$$

$$\begin{aligned}
z_2 h_1 w_1^- w_2^+ &= -\frac{i}{2}(\lambda_6 - \lambda_4) \quad , & z_2 z_2 w_2^+ w_2^- &= 2(\lambda_2 + \lambda_3) \\
z_2 z_2 h_2 h_2 &= 2(\lambda_2 + \lambda_3) \quad , & z_2 z_2 z_2 z_2 &= 6(\lambda_2 + \lambda_3) \\
z_1 z_1 z_1 z_1 &= 6(\lambda_1 + \lambda_3) \quad , & z_2 z_2 z_1 z_1 &= 2\lambda_3 + \lambda_5 \\
z_1 z_1 h_2 h_2 &= 2\lambda_3 + \lambda_6 \quad , & h_1 h_1 z_1 z_1 &= 2(\lambda_1 + \lambda_3) \\
h_1 h_1 z_2 z_2 &= 2\lambda_3 + \lambda_6 \quad , & z_1 h_2 w_1^+ w_2^- &= -\frac{i}{2}(\lambda_6 - \lambda_4) \\
z_1 h_2 w_1^- w_2^+ &= \frac{i}{2}(\lambda_6 - \lambda_4) \quad , & z_1 z_1 w_1^+ w_1^- &= 2(\lambda_1 + \lambda_3) \\
z_1 z_1 w_2^- w_2^+ &= 2\lambda_3 + \lambda_4 \quad , & z_1 z_2 h_1 h_2 &= (\lambda_5 - \lambda_6)/2 \\
z_1 z_2 w_1^+ w_2^- &= (\lambda_5 - \lambda_4)/2 & z_1 z_2 w_1^- w_2^+ &= (\lambda_5 - \lambda_4)/2
\end{aligned}$$

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